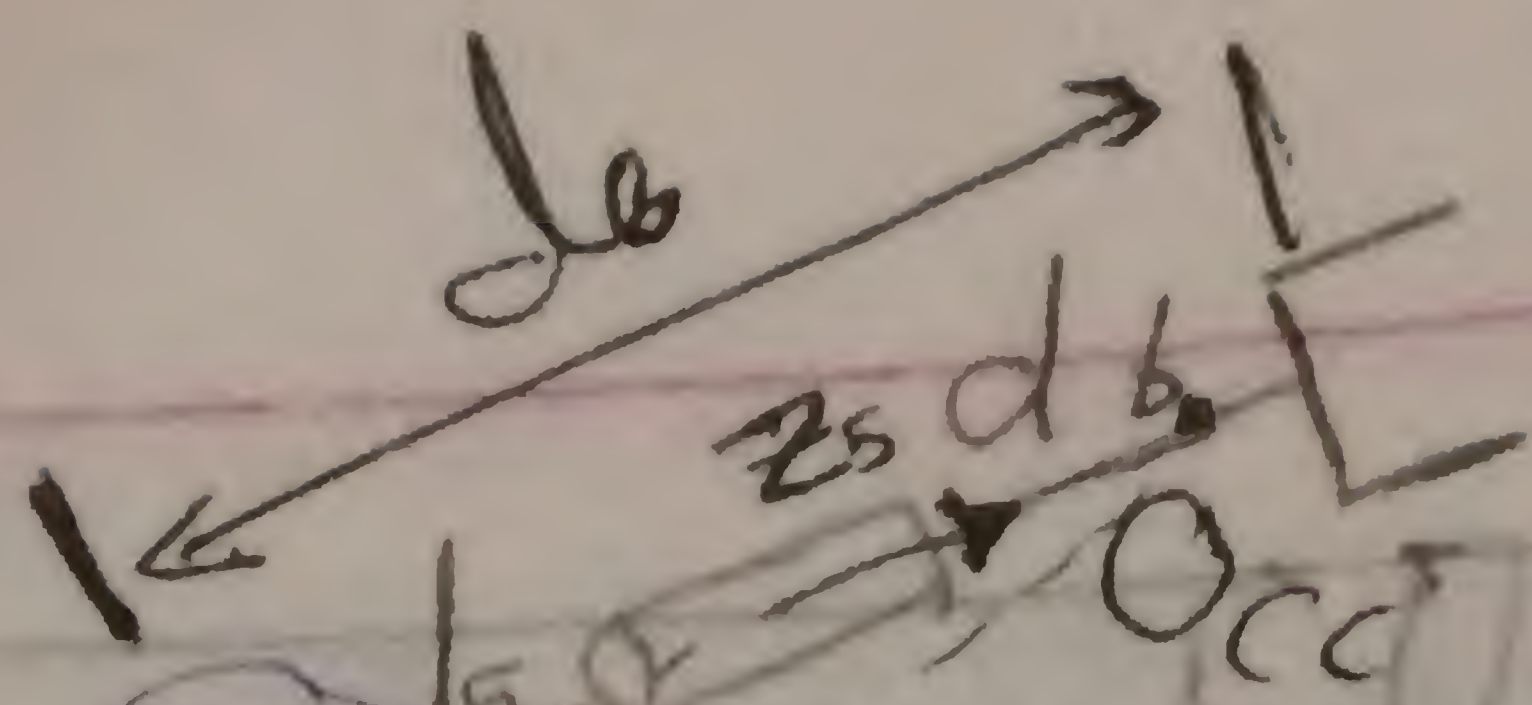


CP 18

see



Robotics
دراسة رياضية
الهندسة، الفيزياء

spherical wrist

3 Angle 3 Axis
for 3 first joint

وكانت هي 3
2 vector 3 axis

xc

elbow manipulator

yc

closed form = Kinematic Decoupling

Steps →

* given: O_{EE} position of end effector, rotation

base
 $R_{EE} \rightarrow I_{EE}$

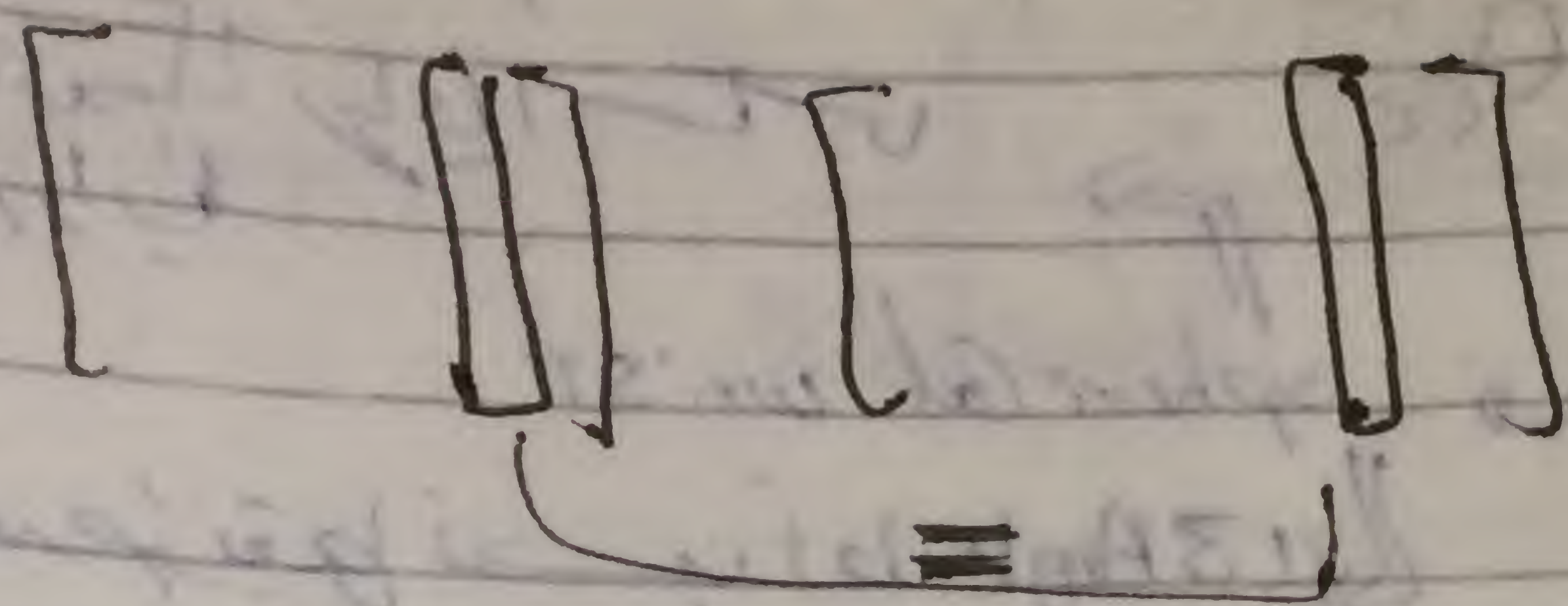
* Required: q_k "ex: elbow $\Rightarrow \theta_1 \rightarrow \theta_6$ "

* Algorithm:

1. $O_C = O_{EE}$ "given" - wrist center.

$$\begin{bmatrix} C30 & -S30 & 0 \\ S30 & C30 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} C20 & -S20 & 0 \\ S20 & C20 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C50 & -S50 & 0 \\ S50 & C50 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_5 R_{EE}



لأن الصير لا يتغير
هذه جميع الأجزاء المختلفة

$$O_C = O_{EE} - d_b R_{EE} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

distance between wrist center and EE

2- From Trigonometric eqn

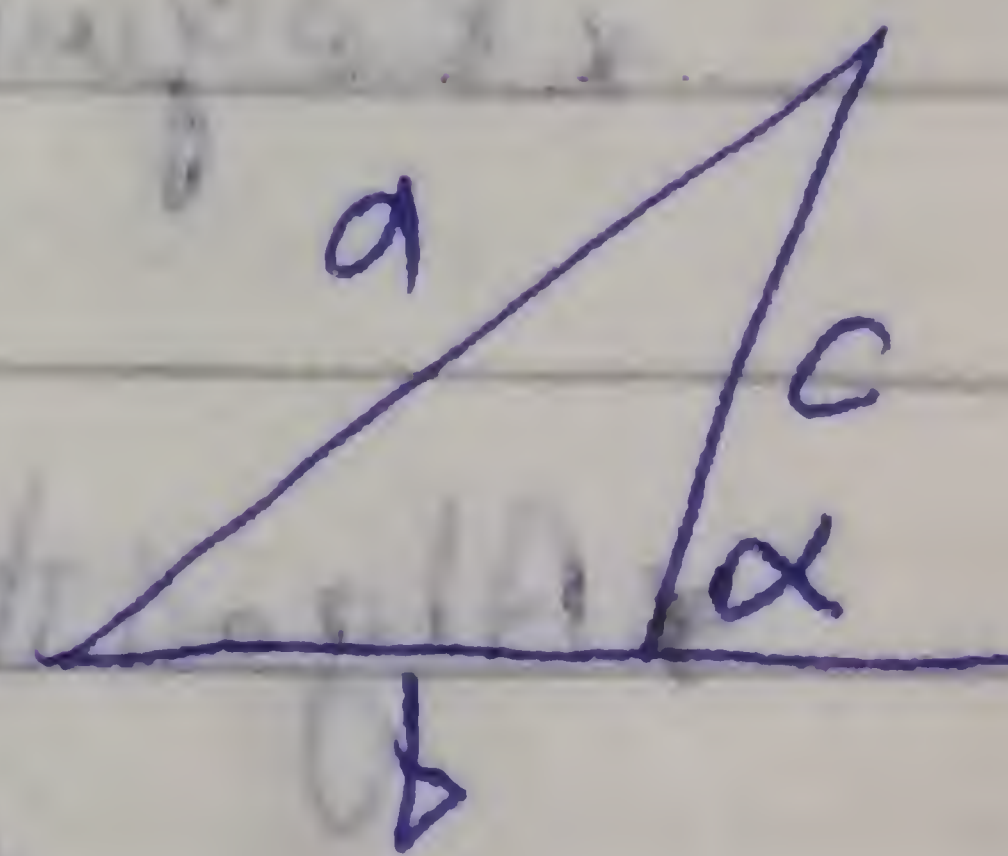
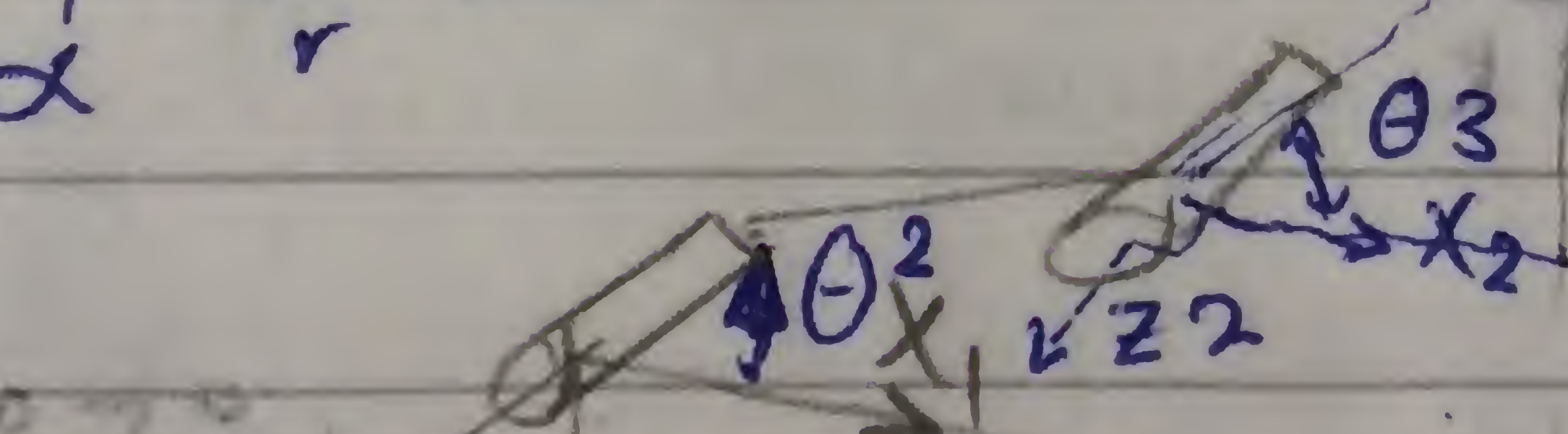
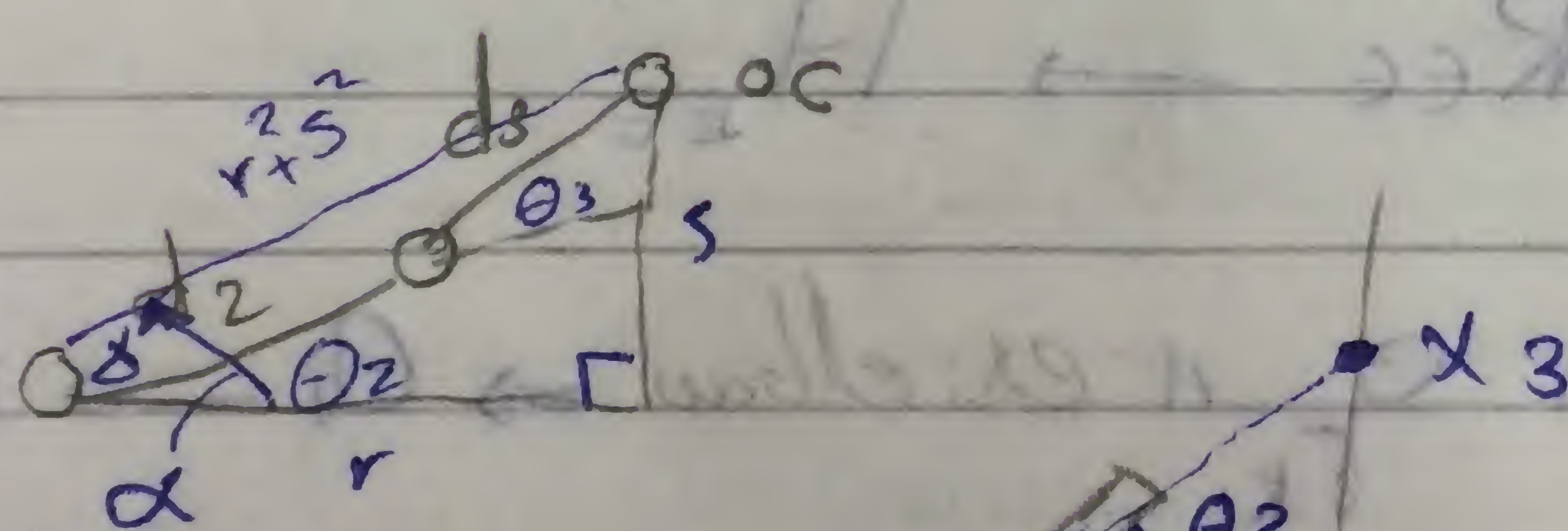
$$\theta_1 = P_1(O_C)$$

$$\theta_2 = P_2(O_C)$$

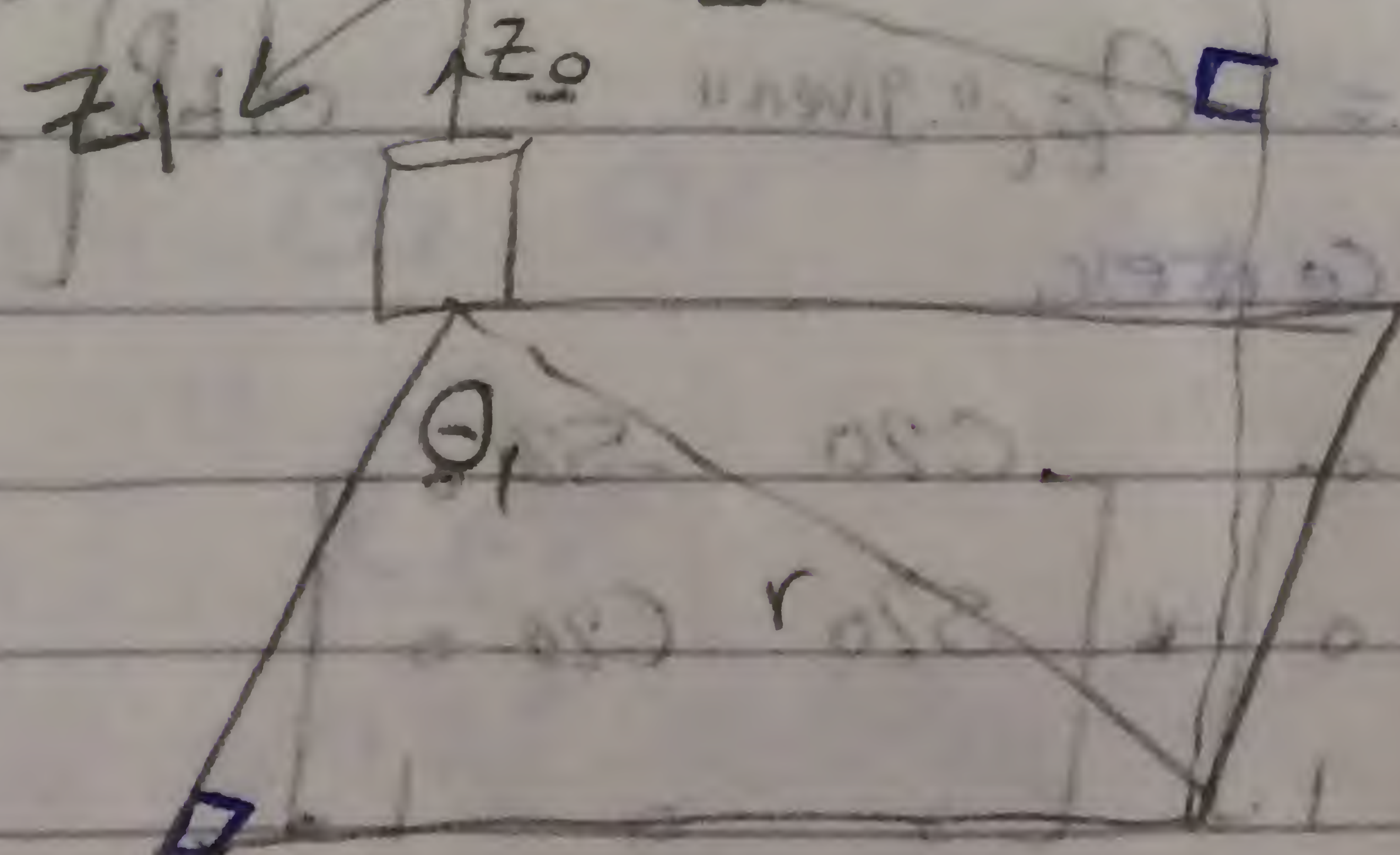
$$\theta_3 = P_3(O_C)$$

كأنه نقطة معرفة x_c, y_c, z_c

is the point where the wrist center is



$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$



x_0

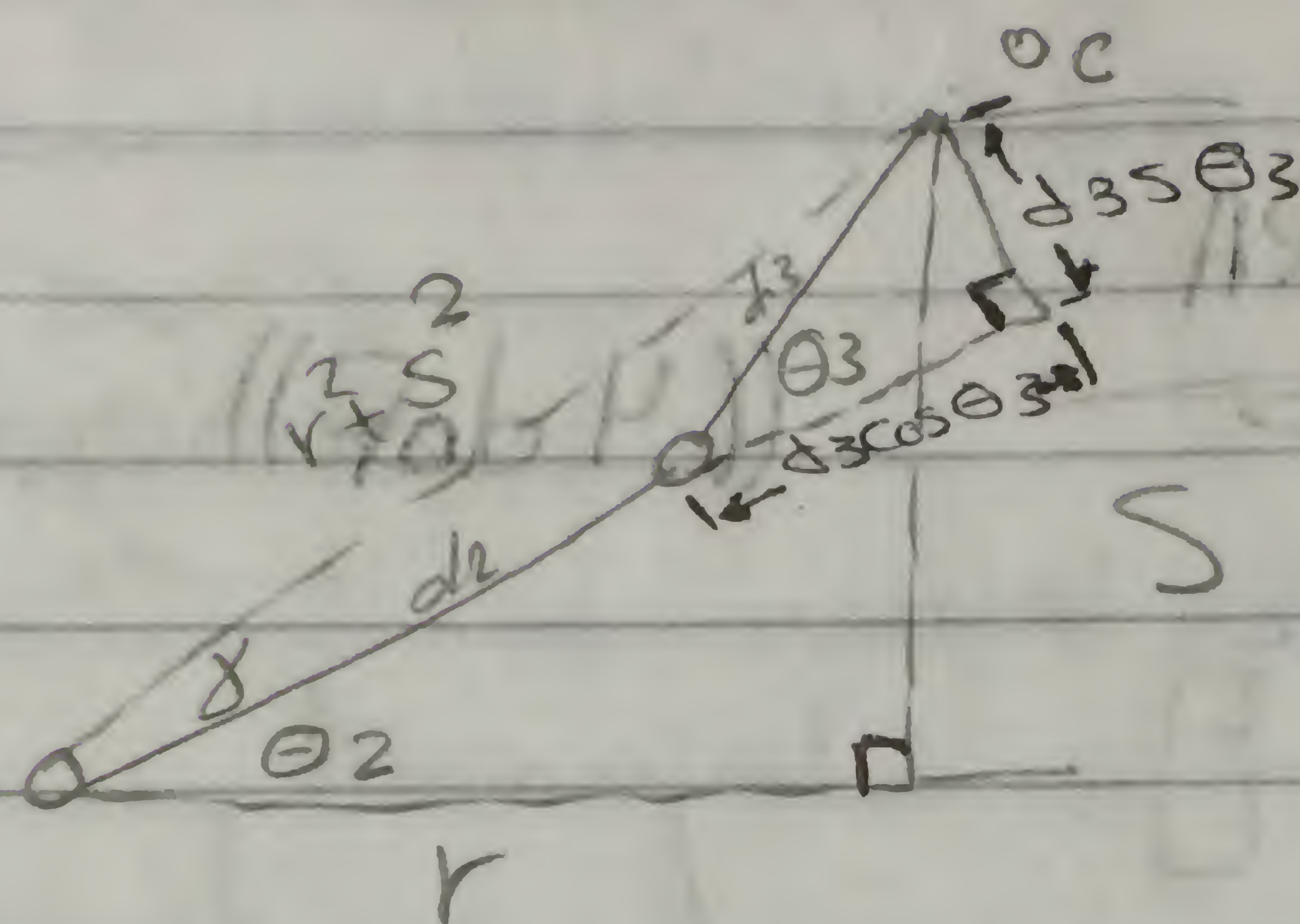
$$\theta_1 = \tan^{-1} \left(\frac{y_c}{x_c} \right)$$

$$\theta_3 = \cos^{-1} \left(\frac{d_2^2 + d_3^2 - (x_c^2 + y_c^2 + (z_c - d_1)^2)}{2 \cdot d_2 \cdot d_3} \right)$$

$$\theta_2 =$$

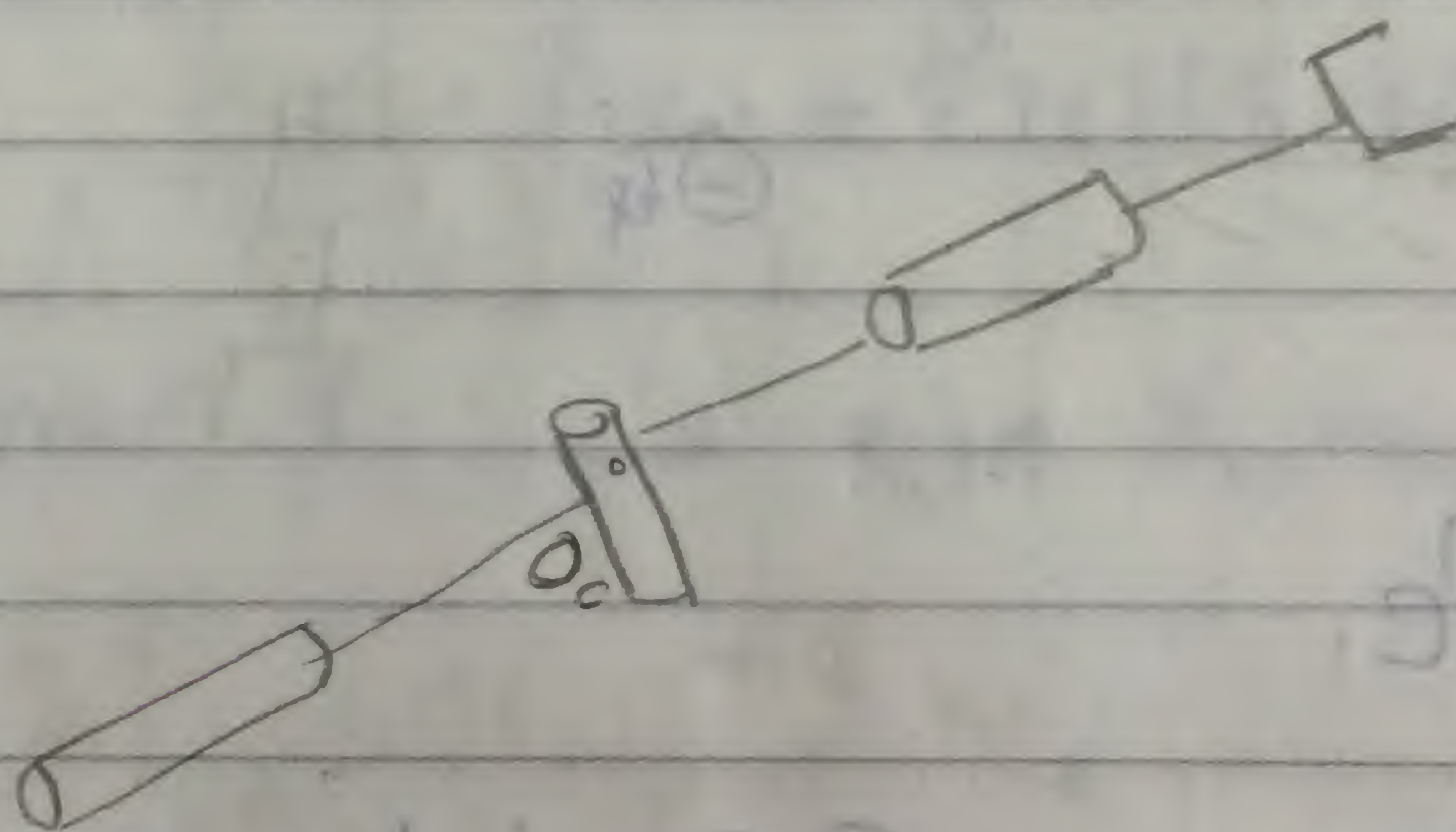
$$\alpha = \tan^{-1} \left(\frac{S}{r} \right)$$

$$\delta = \tan^{-1} \left(\frac{d_3 \sin \theta_3}{d_2 + d_3 \cos \theta_3} \right)$$



3. Orientation.

desired $\rightarrow \theta_4$
 $\rightarrow \theta_5$
 $\rightarrow \theta_6$



$$R_{EE} = {}^0R_3 {}^3R_6$$

0R_3 : from $\theta_1, \theta_2, \theta_3$

$${}^3R_6: [R_3]^{-1} R_{EE}$$

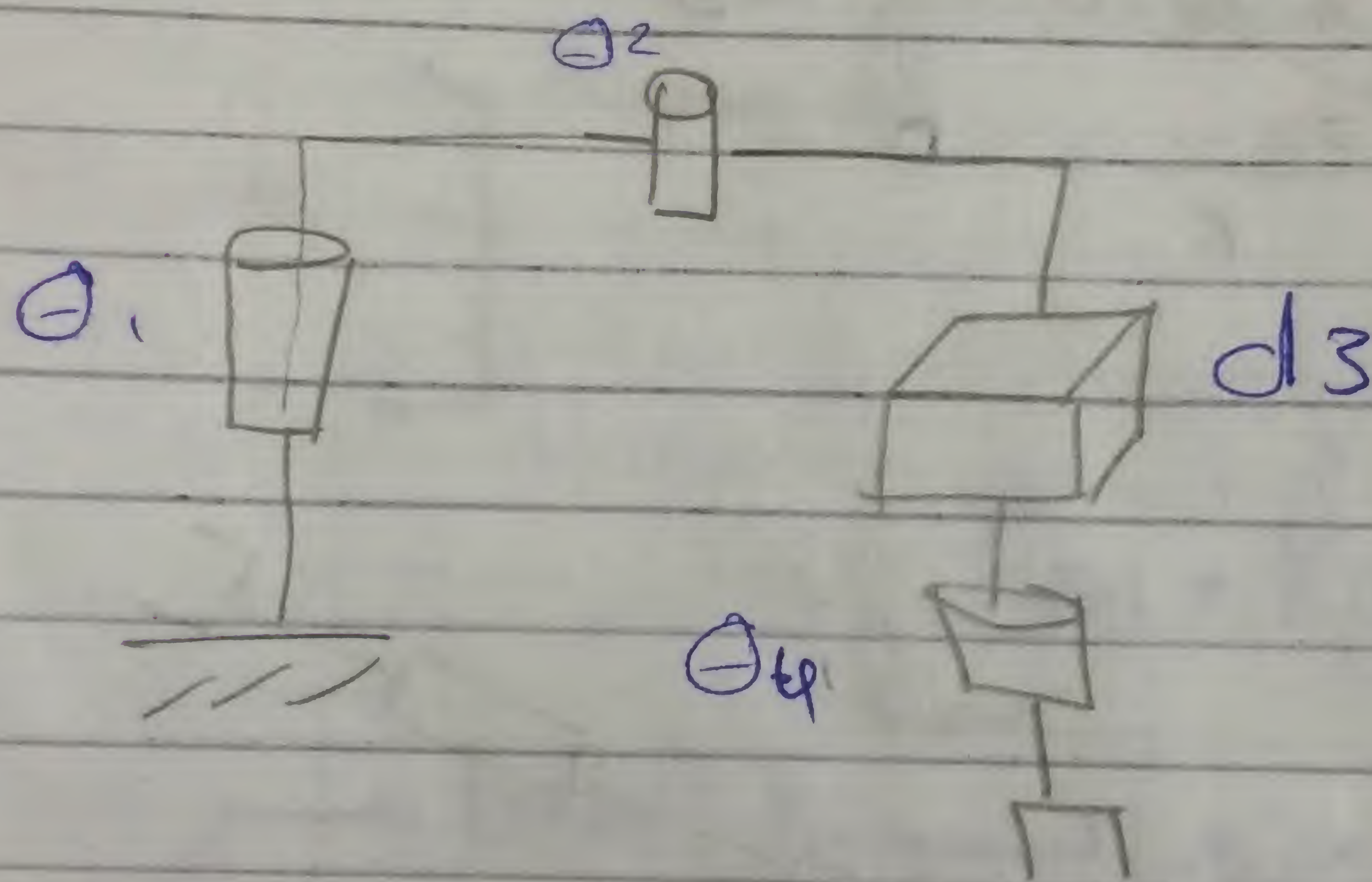
\hookrightarrow spherical wrist orientation \rightarrow Euler Angles.

From 3R_6 using Euler parametrization: $\theta_4, \theta_5, \theta_6$

Euler matrix $\begin{bmatrix} \cos\theta_4 \cos\theta_5 & \sin\theta_4 \cos\theta_5 & -\sin\theta_5 \\ \sin\theta_4 \cos\theta_5 & \cos\theta_4 \cos\theta_5 & \cos\theta_5 \\ \sin\theta_5 & \cos\theta_5 & 0 \end{bmatrix}$
 θ, ϕ, ψ
 ~~$\theta_4, \theta_5, \theta_6$~~

SCARA

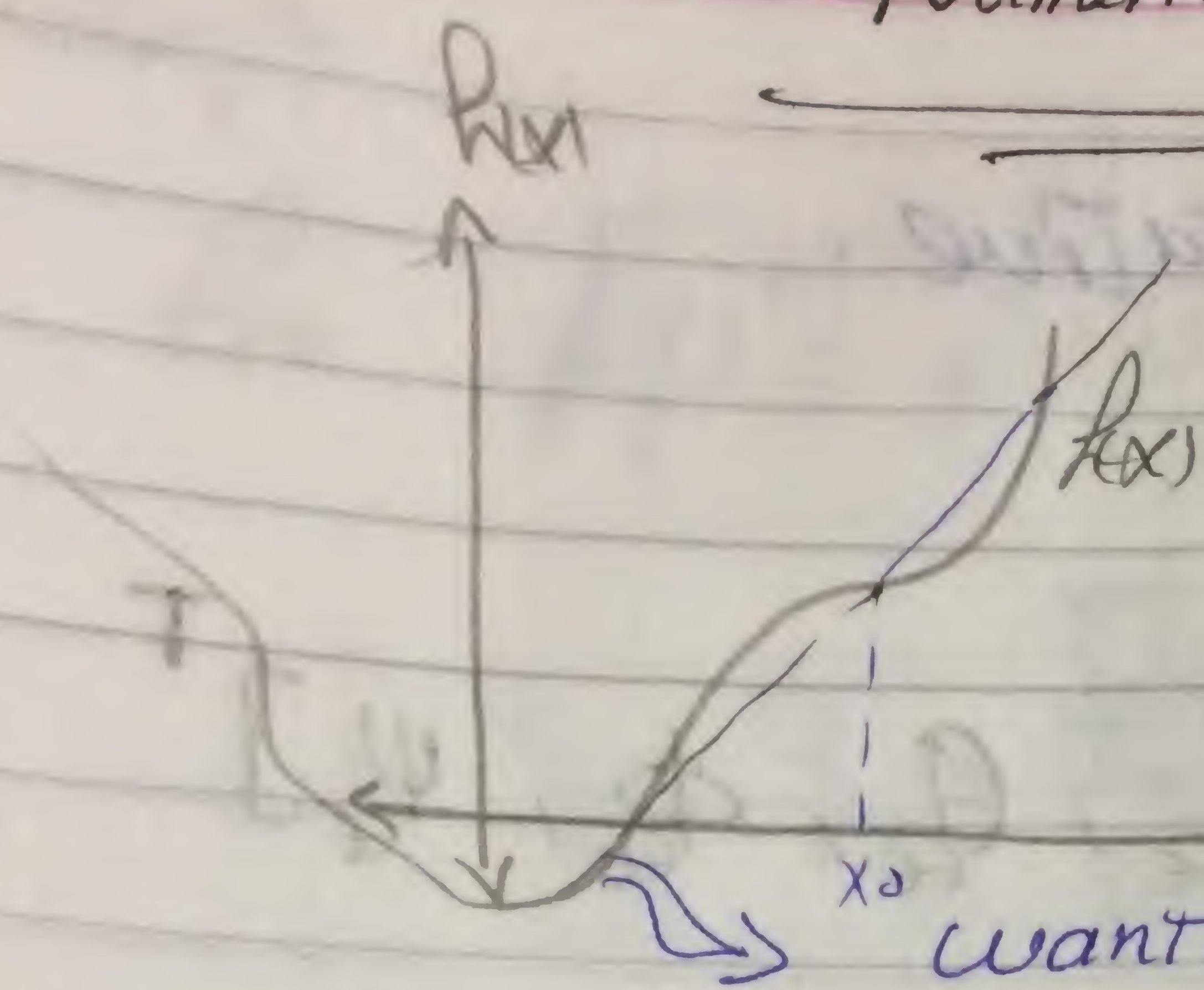
((4 dof))



$$\theta_{ec} = \theta_c$$

Position $\in \mathbb{E}$ independent θ_4

Numerical Methods



linear approximation of $f(x)$
 التقريب الخطي لـ $f(x)$

wanted to obtain x that make $f(x)=0$

Taylor expansion

أي دالة تقدر بتقريبها بـ 2 أو 3 أو 4...

f_{ns} 1st order
 f_{ns} 2nd order
 \vdots
 binomial

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

linear

كل x_0 range جزء error جزء x

كلما بقربنا من x_0 يكون التقدير أكثر دقة

$$f(x) \approx 0$$

linear

$$0 = f(x_0) + f'(x_0)(x - x_0)$$

root

$$-f(x_0) = f'(x_0)(x - x_0)$$

$$x - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

$$x = x_0 - \frac{f(x_0)}{f'(x_0)}$$

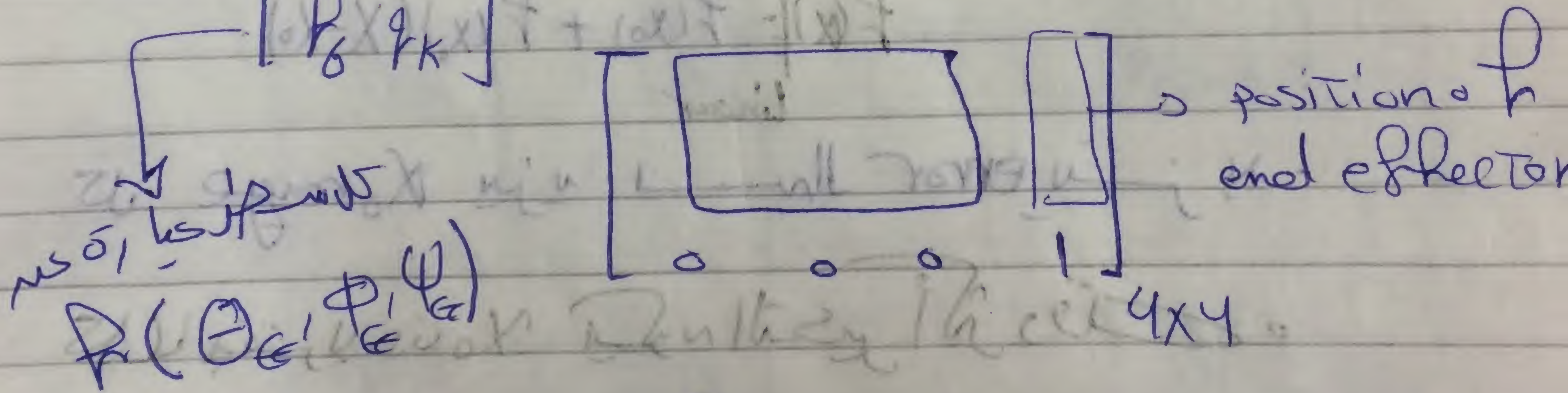
$$X = X_0 - \frac{f(X_0)}{f'(X_0)} \quad \text{iterative.}$$

given vector $g [x_e, y_e, z_e, \theta_e, \phi_e, \psi_e]^T$
desired pose

N (DoF) robot: $\rightarrow i = 1, \dots, N$

$$h(q_k) = \begin{bmatrix} h_1 q_k \\ h_2 q_k \\ \vdots \\ h_6 q_k \end{bmatrix}$$

from forward kinematics.



6 non linear $f_{ns} \rightarrow \psi, \theta, \phi, \psi$
 6/1p required parameters $\rightarrow \vec{r}$
 6/1p 6 parameters of end effector

Steps

Let $F(q) = L(q) - \dot{q} = 0$ at the desired pose

$$q = q_0 - \frac{F(q_0)}{F'(q_0)}$$

$$\frac{\partial F_i}{\partial q} = \frac{\partial F_i}{\partial q_1} + \frac{\partial F_i}{\partial q_2} + \dots + \frac{\partial F_i}{\partial q_n}$$

$$\frac{dF_n}{dq} = \text{multi-variable} + \frac{dF_n}{dq_n}$$

$$\frac{\partial F(q)}{\partial q} = \begin{bmatrix} \frac{\partial F_1}{\partial q} \\ \vdots \\ \frac{\partial F_n}{\partial q} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \dots & \frac{\partial F_1}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_n}{\partial q_1} & \dots & \frac{\partial F_n}{\partial q_n} \end{bmatrix} \begin{bmatrix} F_1(q) \\ F_2(q) \\ \vdots \\ F_n(q) \end{bmatrix}$$

Jacobian

اگر من F^{ns} متغیر واحد

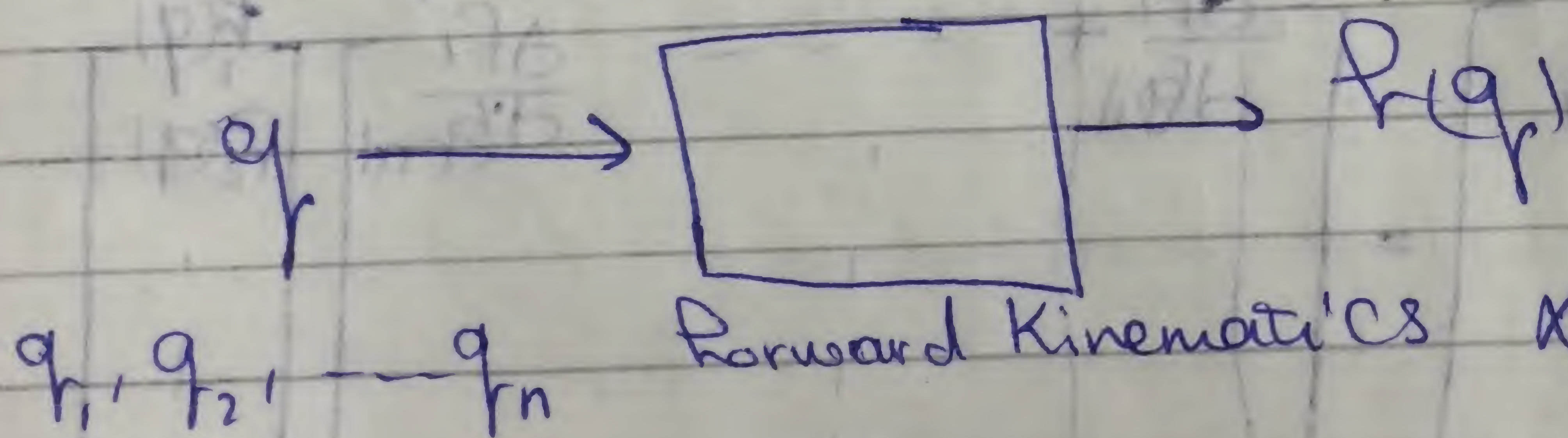
واله واحدی اگر من متغیر

$$q_{n \times 1} = q_{n \times 1} - \underbrace{(F(q))}_{n \times 6} \underbrace{F(q)}_{6 \times 1}$$

only for square matrix \leftarrow inverse

pseudo-inverse.

Artificial intelligence



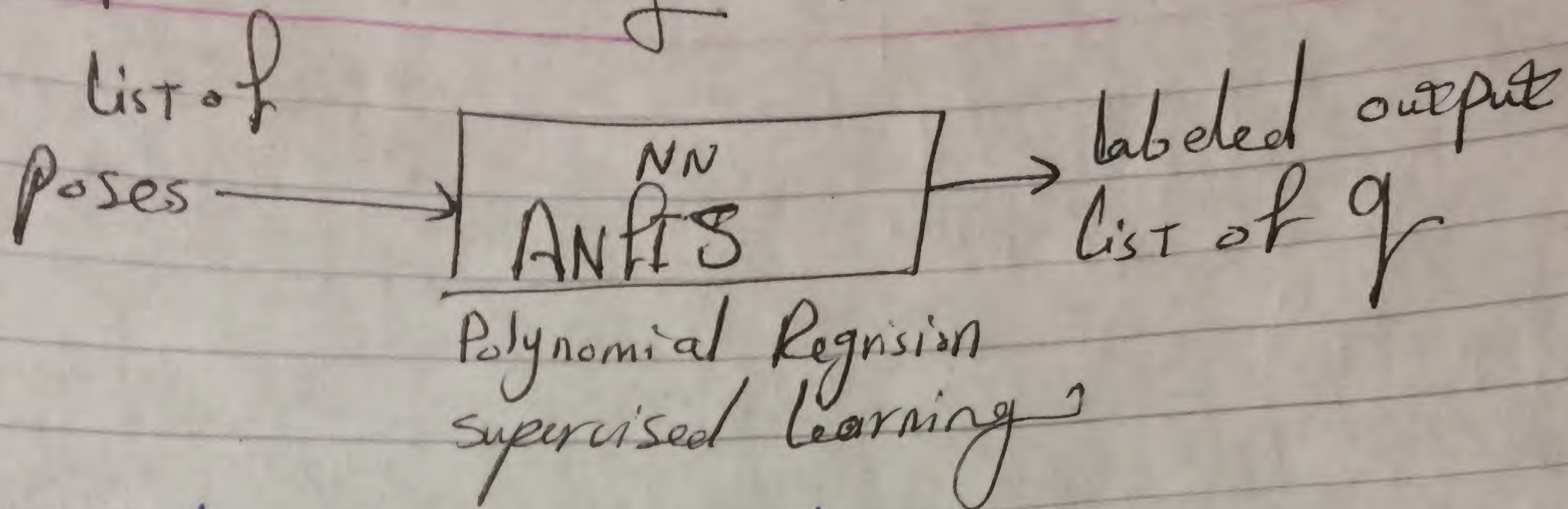
$x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R}, \theta \in \mathbb{R}, \phi \in \mathbb{R}$
pose

Try ranges for q

1. drive Forward Kinematics.
2. Try ranges of q and we obtain same Pose

"dataset"
pose

3. Supervised Learning "PSO"



* we have no direct dataset.

" Trained NN / neural network"

